Problem – 1: Find the value of $\lim\limits_{x \to -4} \dfrac{2x+8}{x^2+x-12}$

Solution:

$$\lim_{x \to -4} \frac{2x+8}{x^2+x-12}$$

$$= \lim_{x \to -4} \frac{2(x+4)}{x^2+4x-3x-12}$$

$$= \lim_{x \to -4} \frac{2(x+4)}{x(x+4)-3(x+4)}$$

$$= \lim_{x \to -4} \frac{2(x+4)}{(x+4)(x-3)}$$

$$= \lim_{x \to -4} \frac{2}{(x-3)}$$

$$= \frac{2}{(-4-3)}$$

$$= -\frac{2}{7} \text{ (Answer)}$$

Problem – 2: Find the value of $\lim\limits_{x \to \infty} \dfrac{2^x-2^{-x}}{2^x+2^{-x}}$

Solution:

$$\lim_{x \to \infty} \frac{2^x-2^{-x}}{2^x+2^{-x}}$$

$$= \lim_{x \to \infty} \frac{2^x\left(1-2^{-2x}\right)}{2^x\left(1+2^{-2x}\right)}$$

$$= \lim_{x \to \infty} \frac{\left(1 - 2^{-2x}\right)}{\left(1 + 2^{-2x}\right)}$$

$$= \frac{1 - 0}{1 + 0}$$

$= 1$ (Answer)

Problem: Find the value of $\lim_{x \to \infty} \frac{x^2}{(x-1)(x-2)}$

Solution:

$$\lim_{x \to \infty} \frac{x^2}{(x-1)(x-2)}$$

$$= \lim_{x \to \infty} \frac{x^2}{x^2\left(1 - \frac{1}{x}\right)\left(1 - \frac{2}{x}\right)}$$

$$= \lim_{x \to \infty} \frac{1}{\left(1 - \frac{1}{x}\right)\left(1 - \frac{2}{x}\right)}$$

$$= \frac{1}{(1-0)\,(1-0)}$$

$= 1$ (Answer)

Problem: $\lim\limits_{x \to 0} \dfrac{\sqrt{1+x}-1}{x}$

Solution:

$$\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x}$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{1+x}-1\right)\left(\sqrt{1+x}-1\right)}{x\left(\sqrt{1+x}+1\right)}$$

$$= \lim_{x \to 0} \frac{1+x-1}{x\left(\sqrt{1+x}+1\right)}$$

$$= \lim_{x \to 0} \frac{x}{x\left(\sqrt{1+x}+1\right)}$$

$$= \lim_{x \to 0} \frac{1}{\left(\sqrt{1+x}+1\right)}$$

$$= \frac{1}{2} \text{(Answer)}$$

Problem: Find the value of $\lim\limits_{x \to 0} \left(\dfrac{\tan x}{x}\right)^{1/x}$

Solution:

$$\lim_{x \to 0} \left(\frac{\tan x}{x}\right)^{1/x}$$

$$= \lim_{x \to 0} \left(\frac{x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \dots\dots}{x}\right)^{1/x}$$

$$= \lim_{x \to 0} \left(1+\frac{1}{3}x^2+\frac{2}{15}x^4+........\right)^{1/x}$$

$$= \lim_{x \to 0} \left(1+\frac{1}{3}x^2\right)^{1/x} \; [\because x \to 0]$$

$$= \lim_{x \to 0} \left\{\left(1+\frac{1}{3}x^2\right)^{3/x^2}\right\}^{x/3}$$

$$= \lim_{x \to 0} e^{x/3}$$

$= e^0$

= 1 (Answer)